# The thermodynamics of quantum information scrambling– A dynamical perspective

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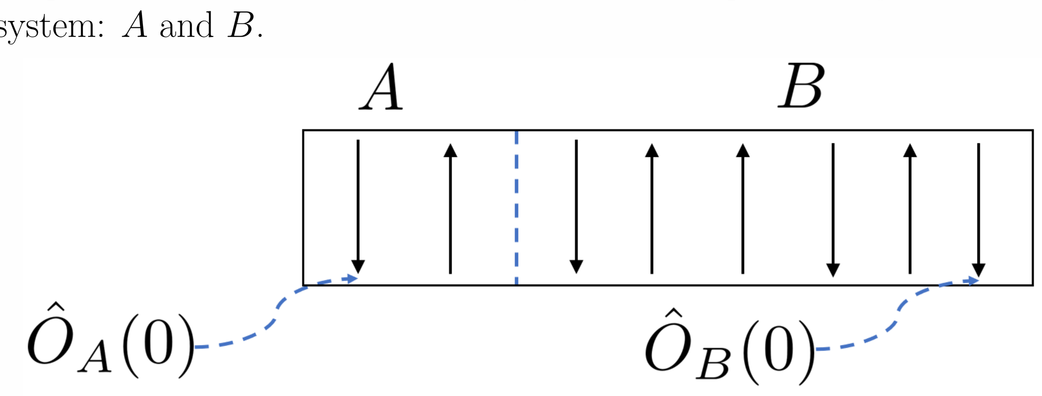
## Abstract

Understanding what happens to a qubit thrown into a black hole is not an easy task; is the information lost to any outside observer or can it be retrieved by local measurements? This problem boils down to studying the scrambling of information over the event horizon, which draws attention to a general class of problems: scrambling of quantum information. In our work, we study the scrambling properties of quantum systems through the mutual information (MI). To this end, we lower bound the MI by the Out-of-Time-Order Correlators (OTOC) and determine its dynamical properties, relating the MI to thermodynamic quantities such as irreversible entropy production. We then illustrate, through examples, the thermodynamic significance of the MI for quantum information scrambling. Thus, as a main result, we establish a clear link between the MI and the OTOC, which enables us to probe the thermodynamics of quantum information scrambling and opens the door for further studies of the thermodynamics of quantum chaotic systems.

## Preliminaries

In order to analyze quantum information scrambling in a closed quantum system S we take two arbitrary partitions of this system: $A$ and $B$.




⇒ The OTOC reads

$$\mathcal{O}(t) = \left\langle \hat{O}_A^\dagger \hat{O}_B^\dagger(t) \hat{O}_A \hat{O}_B(t) \right\rangle,$$

$\hat{O}_A$ and $\hat{O}_B$ are local operators acting on the supports $A$ and $B$ respectively.

⇒ The mutual information $\mathcal{I}$ is

$$\mathcal{I}(t) = S_A(t) + S_B(t) - S_{AB}(0),$$

$S_i = -\mathrm{tr}\{\rho_i \ln(\rho_i)\}$ is the Von Neumann entropy of system $i$ with density matrix $\rho_i$.

## Relating the mutual information and the OTOC

⇒ We adopt the following notation:

$$\int_{\text{Haar}} d\hat{O}_A d\hat{O}_B \left\langle \hat{O}_A \hat{O}_B(t) \hat{O}_A \hat{O}_B(t) \right\rangle = \bar{\mathcal{O}}(t),$$

where the average is taken over the Haar measure of unitaries (on the unitary group U(d)) with

$$\int_{\text{Haar}} dU = 1,$$

$$\int_{\text{Haar}} dU f(U) = \int_{\text{Haar}} dU f(VU) = \int_{\text{Haar}} dU f(UV).$$

For an arbitrary function $f$ and $\forall\, V \in U(d)$ [1, 2].

⇒ We have proven an inequality between the OTOC and the mutual information,

$$(\forall \mathrm{t} \in [0, t_*])\,;\ \mathcal{I}(t) \geq \bar{\mathcal{O}}(0) - \bar{\mathcal{O}}(t),$$

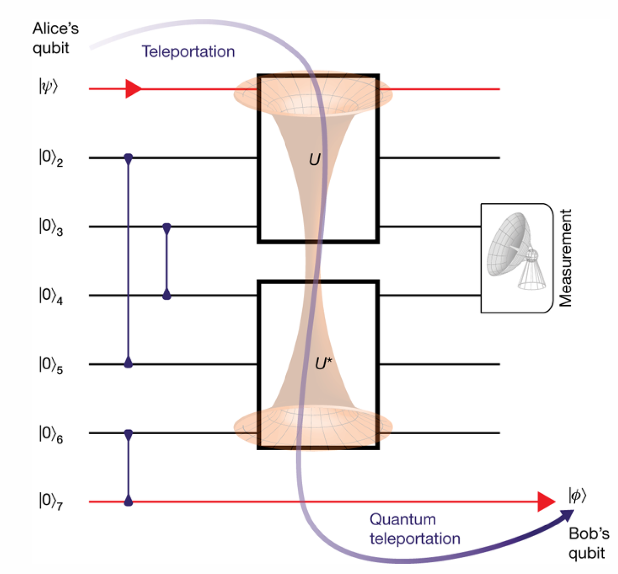
here, $t_*$ is the scrambling time.

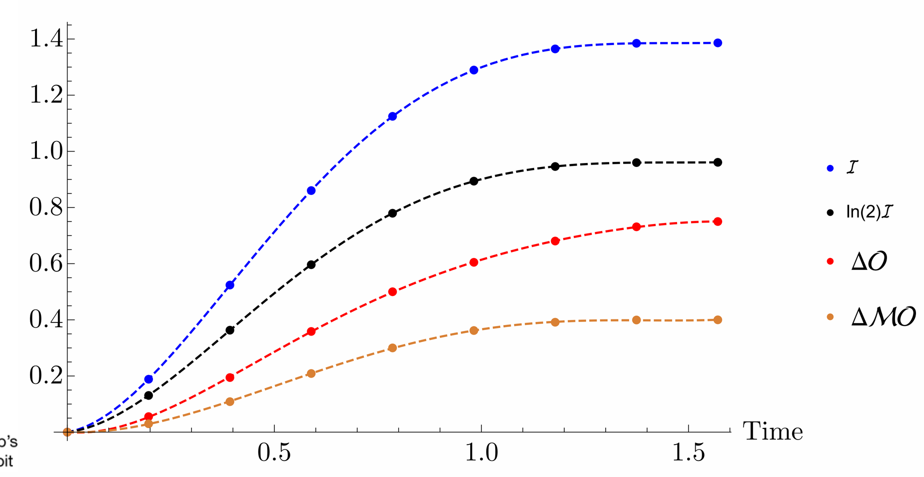
⇒ Problems with the OTOC:

- Choice of operators $\hat{O}_A$ and $\hat{O}_B$.
- Chance of getting flase positives.
- As a correlation function, the OTOC is not directly related to thermodynamic quantities.

## Illustrations

- **Verified quantum information scrambling:**



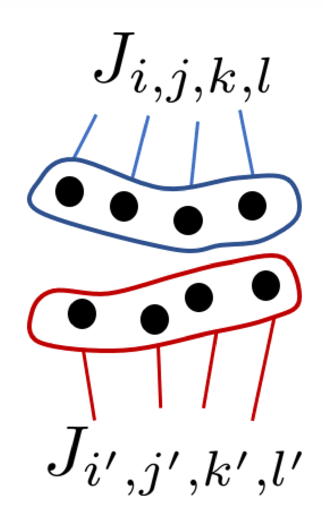

In Ref. [3], the scrambling of information was verified through a teleportation scheme and quantified using a specific average over the OTOC or "modified" OTOC ($\mathcal{MO}$).

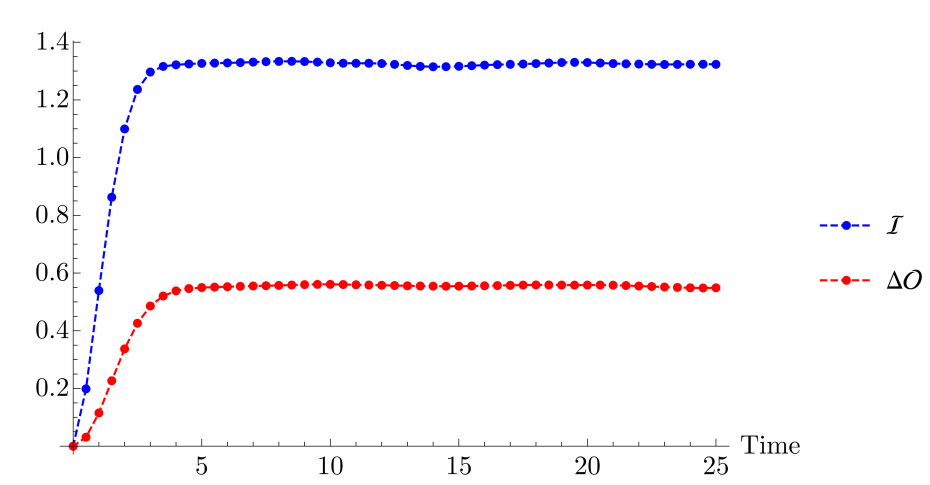
$\mathcal{MO}$ is written as

$$\mathcal{MO}(t) = \sum_{\phi, \hat{O}_\mathrm{P}} \left\langle \hat{O}_1^\dagger \hat{O}_\mathrm{P}^\dagger(t) \hat{O}_1 \hat{O}_\mathrm{P}(t) \right\rangle,$$

such that $\hat{O}_1 \equiv |\psi\rangle\langle\phi|$, and $\hat{O}_\mathrm{P}(t)$ are Pauli matrices evolved by a scrambling unitary in the Heisenberg picture.

- **The SYK model:**




The Sachdev-Ye-Kitaev (SYK) model is a vector model of an exactly solvable quantum chaotic many-body system. The model consists of $N$ interacting Majorana fermions with random interactions involving $q$ of these fermions at each instance ($q$ taken as an even number) [4]. The Hamiltonian of the model reads

$$H = (i)^{\frac{q}{2}} \sum_{1 \leq i_1 < i_2 < \cdots < i_q \leq N} J_{i_1 i_2 \cdots i_q} \psi_{i_1} \psi_{i_2} \cdots \psi_{i_q}.$$

⇒ Remarks on the mutual information:

- $\mathcal{I}(0) = 0$ and $\mathcal{I}_{Max}(t_*) = \min\{d_A, d_B\} \ln(2)$.
- No choice of operators is needed to compute $\mathcal{I}(t)$.
- No false positives in scenarios presented in Ref. [3].

**Mutual information can be directly linked to thermodynamic quantities.**

## The thermodynamics of information scrambling

We can upper bound the rate of change of the mutual information by thermodynamic quantities. We have

$$\dot{\mathcal{I}} \leq \alpha \cdot (\dot{S}_{I_A})^{\frac{1}{2}} + \beta \cdot (\dot{S}_{I_B})^{\frac{1}{2}} + \gamma \cdot |\dot{S}_E|,$$

where $(\alpha, \beta, \gamma)$ depend on the geometry of our subsystems. While $\dot{S}_{I_A}$ and $\dot{S}_{I_B}$ are the stochastic irreversible entropy productions in $A$ and $B$ respectively, $\dot{S}_E$ is the entropy (or information) flow between $A$ and $B$, by direct analogy to terms defined in Ref. [5].

## References

[1] Daniel A Roberts et al.(2017). *Journal of High Energy Physics*, 2017(4):121.
[2] Jordan Cotler et al.(2017). *Journal of High Energy Physics*, 2017(11):48.
[3] Kevin A Landsman et al.(2019). *Nature*, 567(7746):61.
[4] Juan Maldacena et al.(2016). *Physical Review D*, 94(10):106002.
[5] Massimiliano Esposito et al.(2010). *Physical Review E*, 82(1):011143.