Bosons Outperform Fermions: The Thermodynamic Advantage of Symmetry

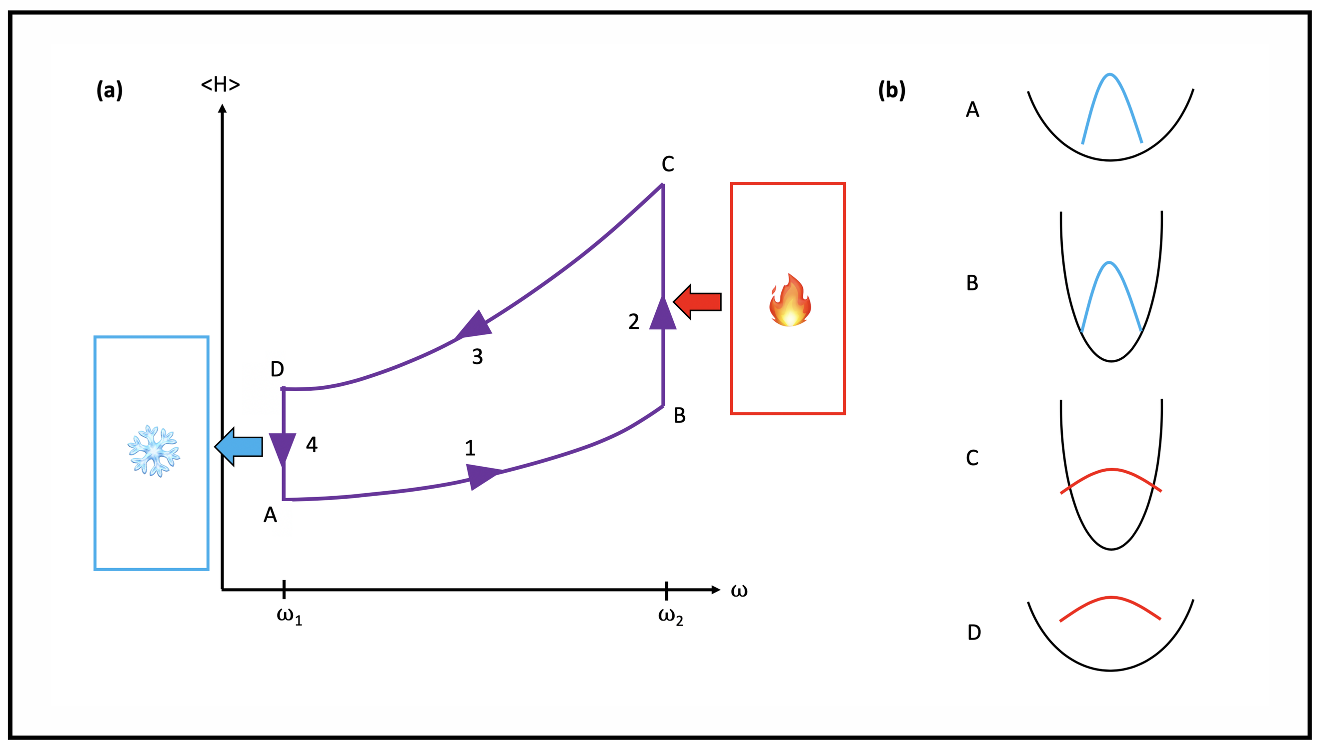
Nathan Myers and Sebastian Deffner

Department of Physics, University of Maryland, Baltimore County, Baltimore, MD 21250, USA

Abstract

The recent miniaturization of heat engines to the nanoscale introduces the possibility of engines that harness quantum resources. The analysis of quantum engines provides important insight into how their efficiency compares to classical analogues and deepens our understanding of thermodynamic mechanisms on the quantum scale. Here we examine a quantum Otto engine with a harmonic working medium consisting of two non-interacting particles to explore the use of wave function symmetry as an accessible quantum resource. We show that a bosonic working medium displays enhanced performance, and a fermionic working medium reduced performance, when compared to independent, distinguishable particles.

The Quantum Otto Engine



⇒ The Otto cycle consists of four strokes: (1) isentropic compression, (2) isochoric heating, (3) isentropic expansion, and (4) isochoric cooling.

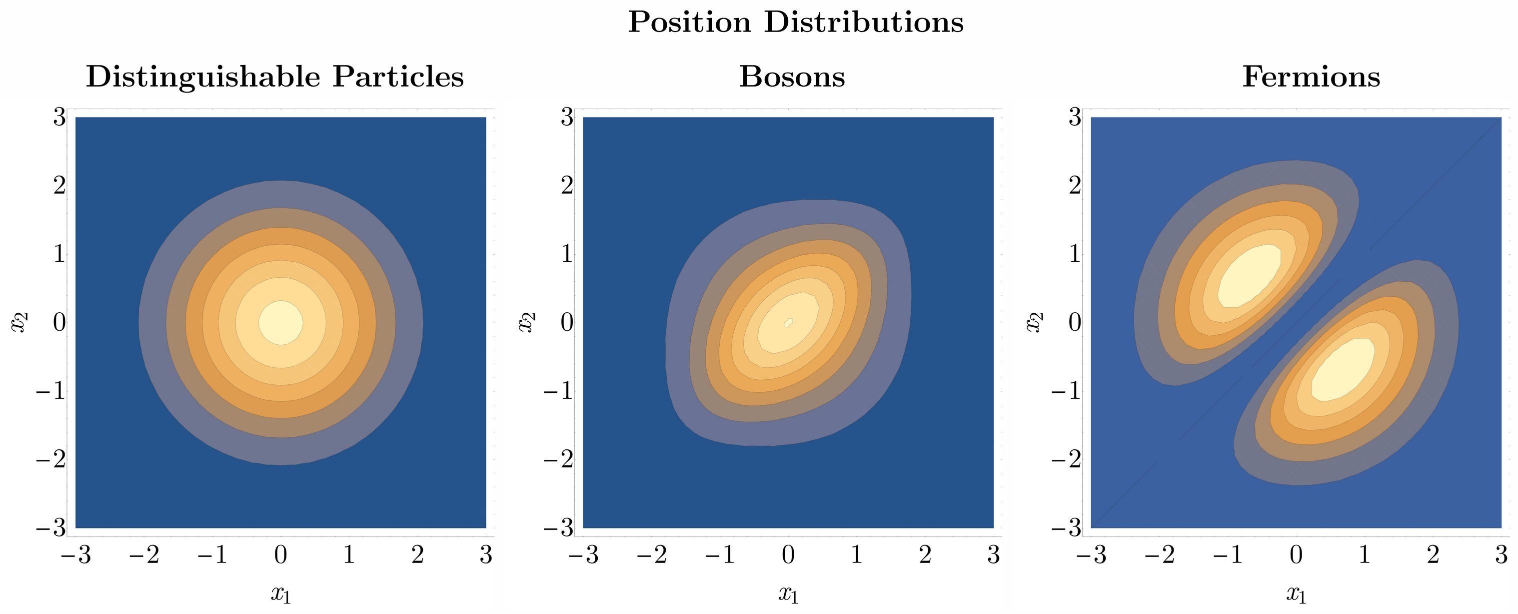
⇒ In a harmonic quantum engine, the isentropic strokes correspond to varying the oscillator frequency and the isochoric strokes correspond to changing the temperature (energy) while holding frequency constant.

⇒ Efficiency and power output in an Otto engine are totally characterized by changes in internal energy. This is something we know how to calculate for quantum systems - the expectation value of the Hamiltonian!

⇒ Here we examine a non-interacting two particle working medium described by the Hamiltonian:

$$H = \frac{p_1^2+p_2^2}{2m} + \frac{1}{2}m\omega^2(x_1^2+x_2^2)$$

Bosons and Fermions: Useful Symmetry?



⇒ Differences in behavior between fermions and bosons is governed by wave function symmetry.

$$\Psi_{n_1,n_2}(x_1,x_2) = \frac{1}{\sqrt{2}}[\psi_{n_1}(x_1)\psi_{n_2}(x_2) \pm \psi_{n_1}(x_2)\psi_{n_2}(x_1)]$$

⇒ This results in an effective repulsion between fermions, and effective attraction between bosons - can we leverage this to extract additional work from the system?

⇒ In a thermal state the two particle system is described by the density matrix:

$$\rho(x_1,x_2,y_1,y_2) = \frac{1}{Z}\frac{m\omega}{2\pi\hbar\sinh(\beta\hbar\omega)}\Big[e^{-\frac{m\omega}{4\hbar}\left(((x_1+y_1)^2+(x_2+y_2)^2)\tanh(\beta\hbar\omega)+((x_1-y_1)^2+(x_2-y_2)^2)\coth(\beta\hbar\omega)\right)}$$
$$\pm e^{-\frac{m\omega}{4\hbar}\left(((x_2+y_1)^2+(x_1+y_2)^2)\tanh(\beta\hbar\omega)+((x_2-y_1)^2+(x_1-y_2)^2)\coth(\beta\hbar\omega)\right)}\Big]$$

Evolving the State

⇒ To determine efficiency and power we must understand how the density matrix for our system evolves in time. We can describe this using an evolution operator:

$$\rho_t(x_1,x_2,y_1,y_2) = \int \mathrm{d}x_1^o \int \mathrm{d}x_2^o \int \mathrm{d}y_1^o \int \mathrm{d}y_2^o\, U(x_1,x_1^o,x_2,x_2^o)\rho(x_1^o,x_2^o,y_1^o,y_2^o)U^\dagger(y_1,y_1^o,y_2,y_2^o)$$

⇒ We can construct the two particle evolution operator from the known single particle evolution operators. Note that here we again see the role of the wave function symmetry:

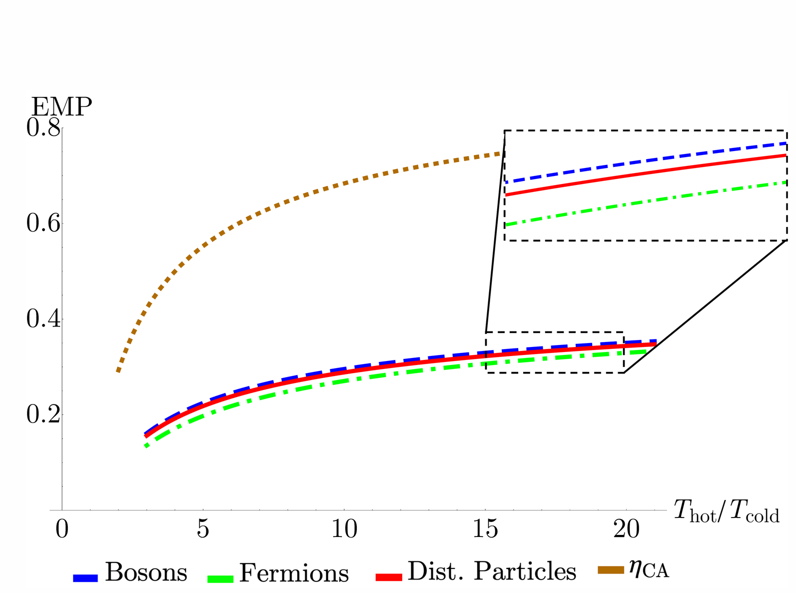
$$\langle n_1 n_2| U_2 |n_1^0 n_2^0\rangle = \frac{1}{2}\left[\langle n_1| U_1 |n_1^0\rangle \langle n_2| U_1 |n_2^0\rangle \pm \langle n_1| U_1 |n_2^0\rangle \langle n_2| U_1 |n_1^0\rangle\right]$$

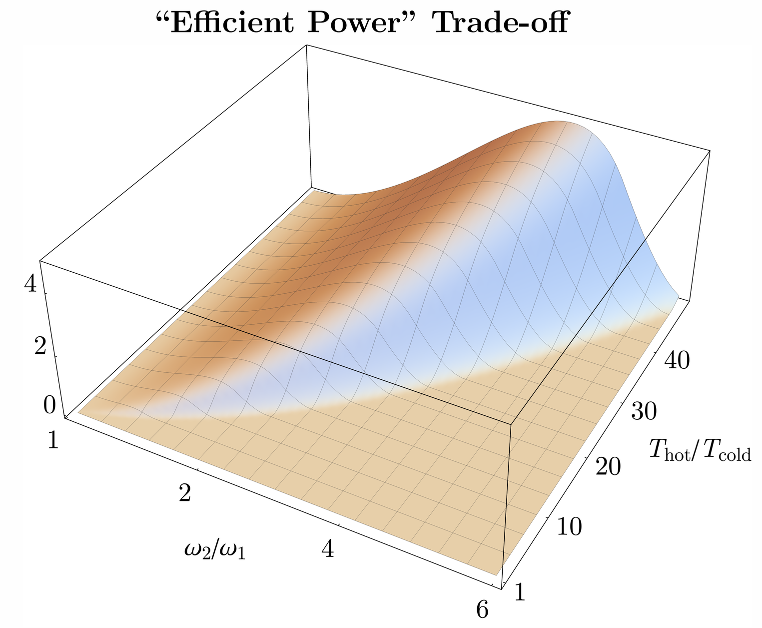
Efficiency and Power

⇒ There is an inherent trade-off between efficiency and power. Maximizing efficiency demands an infinite cycle time, and thus results in zero power.

⇒ For practical measures of engine performance we examine the ***efficiency at maximum power*** (EMP).

⇒ The *efficient power* (product of power and efficiency) quantifies power gained per corresponding reduction in efficiency.





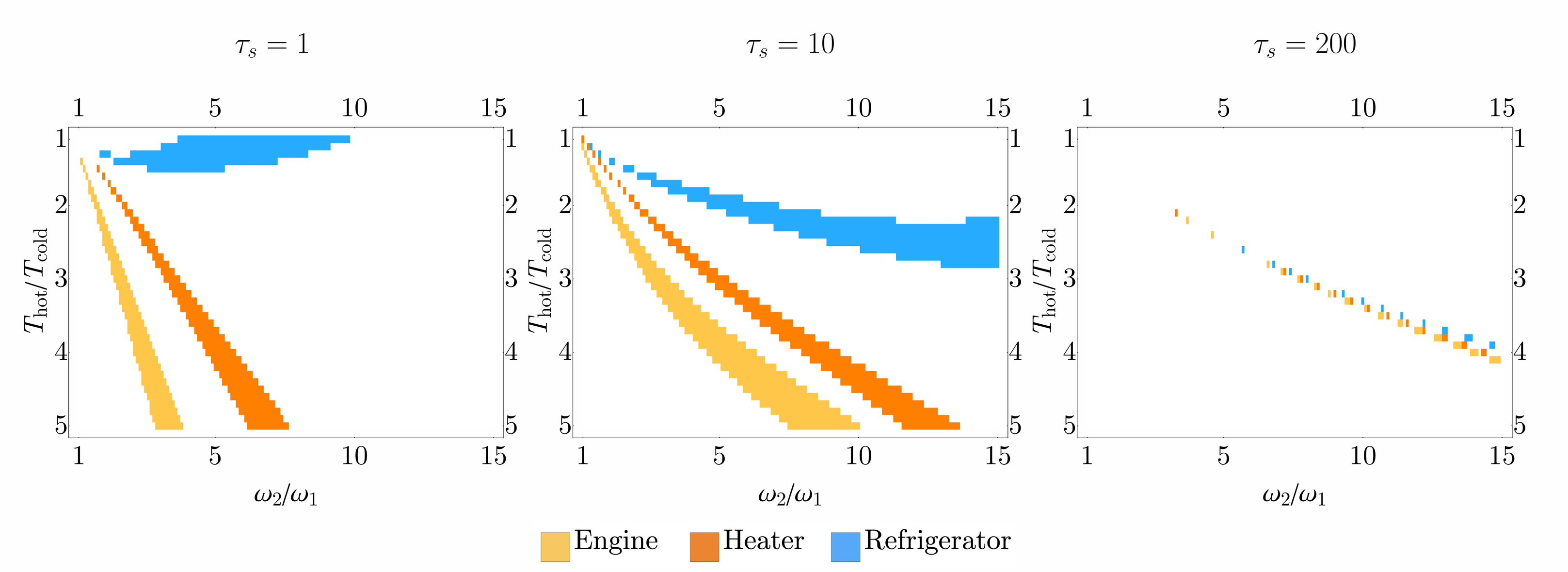
⇒ For the case of an instantaneous frequency switch we see an increase in EMP for the bosonic working medium, and a decrease for the fermionic medium (in comparison to distinguishable particles). The top line gives the Curzon-Ahlborn EMP (classical, quasistatic EMP limit).

⇒ Examining the difference between the boson and fermion efficient power we see the bosons demonstrate a superior trade-off.

Operational Parameter Regimes

⇒ An additional advantage for the bosonic medium manifests in terms of the range of parameters under which the cycle still functions as the desired thermal machine (engine, heater, or refrigerator).

⇒ The plots below show the regimes under which the bosons function as an engine, refrigerator, and one type of heater while the fermions do not. A linear frequency switch is used with stroke time τ_s. Note in large τ_s limit (classically quasistatic) symmetry effects vanish.



Reference

N. M. Myers and S. Deffner. *Phys. Rev. E* 101, 012110 (2020).