Violating Bell's Inequality Using a Number State and a Beam Splitter

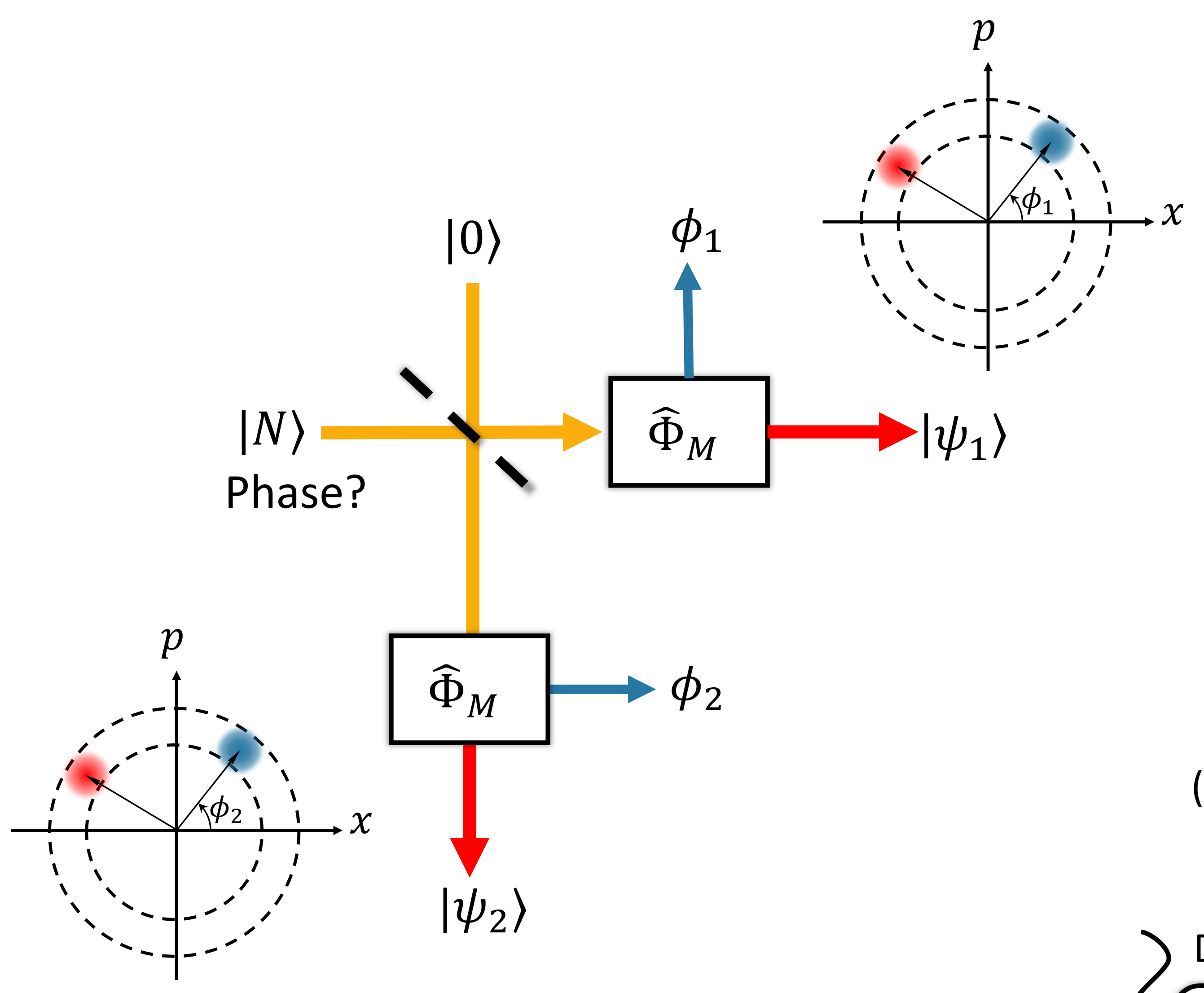
S.U. Shringarpure and J.D. Franson

Physics Department, University of Maryland Baltimore County

We show that a **photon number state** incident on a beam splitter will create **phase-entangled states**

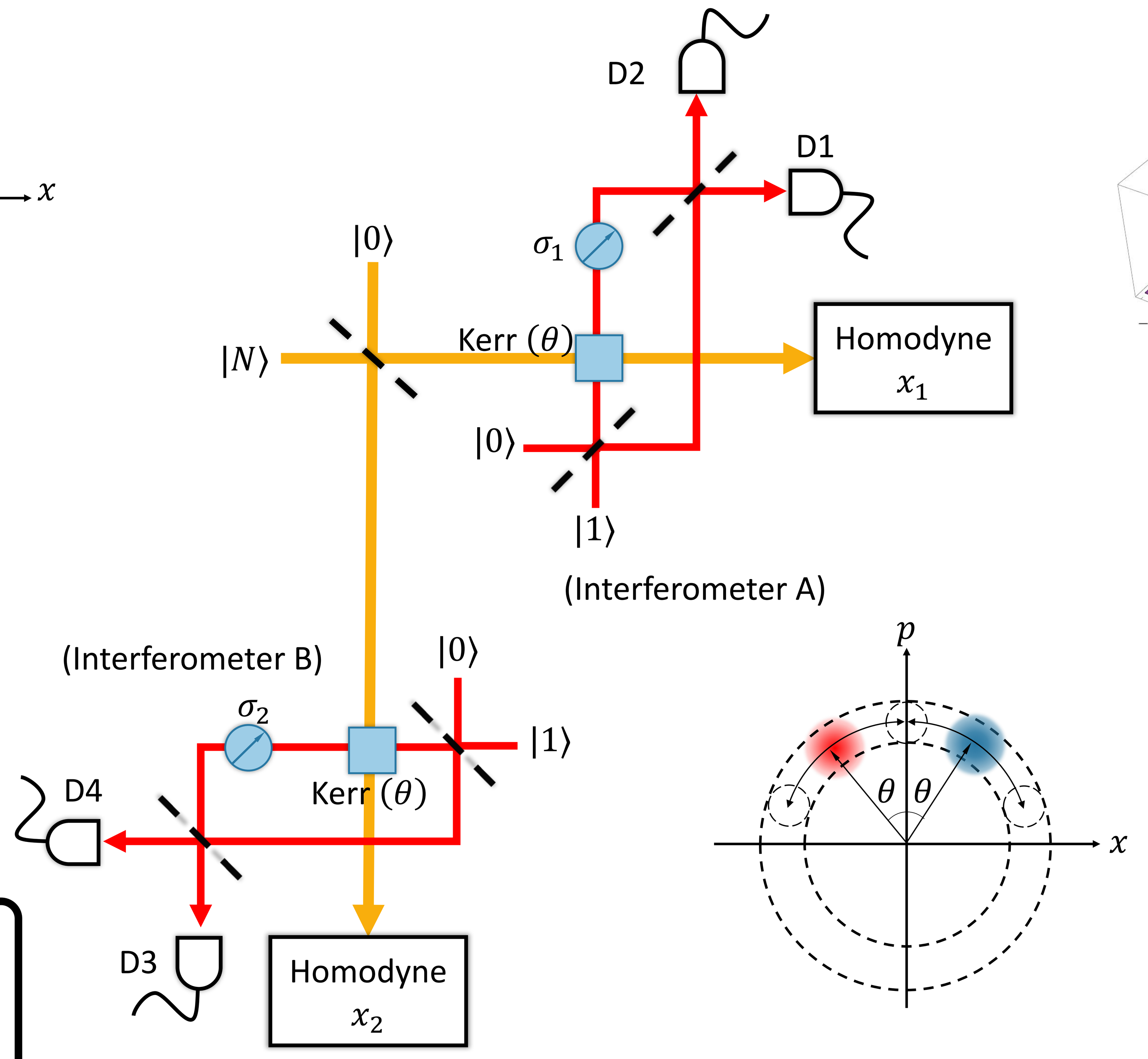
These entangled states **violate Bell's inequality** and this approach may have practical applications in **quantum communications**

Introduction



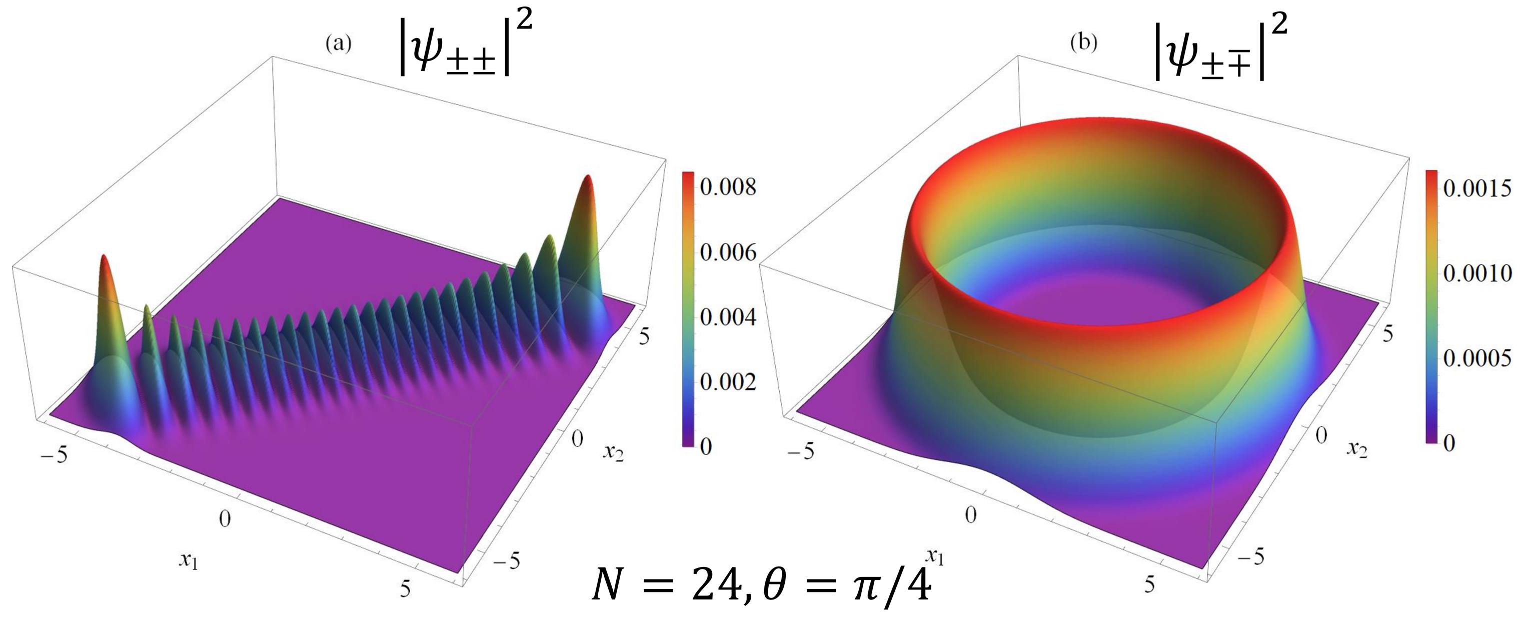
- **Operator $\hat{\Phi}_M$ represents a phase measurement** performed by splitting off a small fraction of the field and using it to measure phases ϕ_i
- Measurement in path 1 will collapse the outputs $|\psi_i\rangle$ to **approximate coherent states** with same ϕ [1]
- Thus, number state can be viewed as **having a specific but unknown phase** that is transferred to both output beams creating a phase-entangled state
- Constructing a measurement which cannot distinguish coherent states with two different phases, should lead to **interference**
- As these phases are entangled, the interference must depend on nonlocal phase shifts we introduce

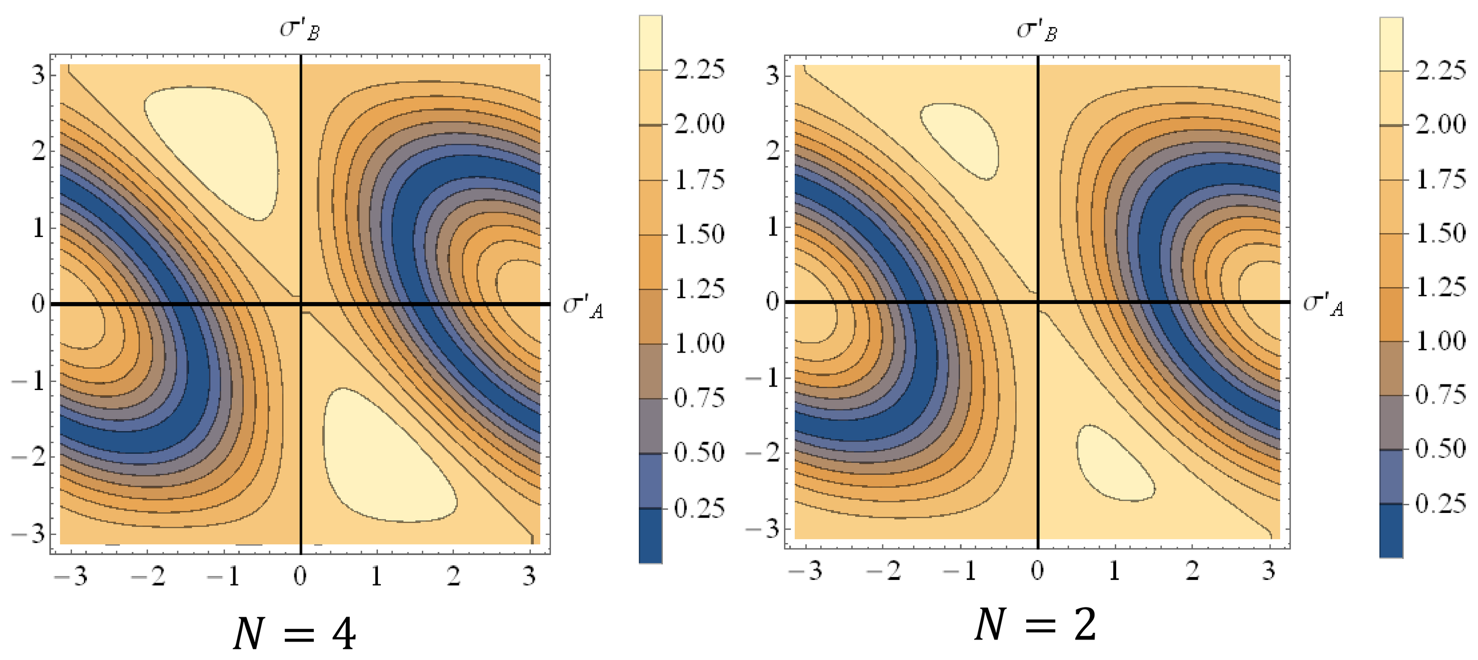
Setup



- Nonlocal quantum interference is produced by applying a phase shift of θ to the two output modes of the beam splitter using two single-photon interferometers containing **Kerr medium** in one path
- Postselection based on homodyne measurements will give a probability amplitude for a successful outcome that is a superposition of terms corresponding to the **two possible values of θ**
- Thus, four interfering amplitudes correspond to **four combinations of paths** single photons can take

Results





- First, we postselect on the outcomes that D1 and D3 detect a photon. In further postselecting on the outcomes of the homodyne measuremens $x_1 = x_2 = \sqrt{N}$, only ψ_{++} and ψ_{--} amplitudes interfere which are controlled by σ_i giving **nonlocal interference** $\sim \cos^2(\sigma_1 + \sigma_2)/2$
- Next, we construct CHSH Bell inequality where we consider two 'settings' for the measurement devices as two different values for σ_i ($\sigma_1 = \sigma_A/\sigma_A'$ and $\sigma_2 = \sigma_B/\sigma_B'$) and the two 'outcomes' ($a = \pm 1$ and $b = \pm 1$) as whether detectors D1/D2 and D3/D4 detect photons. Postselection on the outcome of homodyne measurements $x_1 = x_2 = \sqrt{N}$ is made before choosing the settings in this case.
- **CHSH Bell inequality**:
$$S = \langle ab \rangle + \langle ab' \rangle + \langle a'b \rangle - \langle a'b' \rangle \leq 2$$
- We chose $\sigma_A = 0, \sigma_B = \pi$ and a Kerr effect of $\theta = \pi/4$. A result of $S > 2$ (a violation of CHSH Bell inequality) was observed for certain range of values of σ_A' and σ_B', even for number states as low as 2

Conclusions

- Number state which is produces phase-entangled states after passing through a beam splitter. Measurement of phase in one mode fixes the phase in the other
- This entanglement can be used for quantum communication applications like QKD. Violation of Bell' inequality is a first step in this direction

Acknowledgements

This work was supported in part by the **National Science Foundation** under grant number PHY-1802472

References

[1] J. D. Franson, Phys. Rev. A 49, 3221 (1994).
[2] S.U. Shringarpure and J.D. Franson, arXiv: 1911.02468.